BER Analysis of STBC with Packet Combining in MIMO Rayleigh Fading Channels: LLR-Based Approach

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***Abstract*— Adopting a log-likelihood ratio (LLR) based approach, we analyze the bit error rate (BER) performance of orthogonal space-time block coding (STBC) using M-ary quadrature amplitude modulation (M-QAM) along with packet combining triggered by automatic repeat request (ARQ) retransmission over multiple-input multiple-output (MIMO) Rayleigh fading channels. Specifically, considering the 16-QAM case of study, we provide an exact formula for the aggregate LLR distribution in the case the STBC codeword can be transmitted twice, and derive an exact expression for the BER. For higher number of retransmissions, an approximation of the error function, erf(.), is used to derive the LLR distributions and the system's ensuing BER. The proposed BER analytical model is validated through simulations considering transmission over additive white Gaussian noise (AWGN) and MIMO Rayleigh fading channels, for different STBC mappings, different values of combined transmissions, and possible constellation rearrangement (CoRe).**

I. INTRODUCTION

Recently, a lot of efforts were dedicated to the study of multiple-input multiple-output (MIMO) systems, both to evaluate the link-level performances and to present coding techniques suitable for the MIMO technology. One of the most simple and attractive coding techniques for the MIMO configuration is space-time block coding (STBC) [1], [2]. In MIMO systems, the use of STBC in conjunction with link adaptation techniques has been shown to allow one to cope with the varying nature of the propagation conditions to optimally exploit the available capacity of the channel in order to maximize throughput and satisfy delay and error rate requirements [3].

Link adaptation can be performed in various forms, for instance, in the form of automatic repeat request (ARQ) retransmission implemented at the link layer. In its simplest form, the ARQ protocol is of type I (ARQ-I) where each packet retransmission carries the same information as the original transmission [4]. ARQ-I is considered for future high-speed wireless networks, and is already adopted in systems such as High-Speed Downlink Packet Access (HSDPA) [5]. To reduce the delay effects of the protocol, the latter can be implemented in a truncated form wherein, upon reception of a negative acknowledgement (NACK), retransmissions can continue until the packet is correctly received or a preset maximum number of retransmissions is attained. Mapping diversity, in the form of constellation rearrangement (CoRe) [5], can also be used to improve the transmission reliability. The performance of ARQ-I protocol can further be enhanced through simple processing at the receiver. Indeed, instead of discarding packets that are received in error, packet combining can be implemented at the receiver.

In this paper, we consider a MIMO STBC system employing M-ary quadrature amplitude modulation (M-QAM) and implementing ARQ-I with packet combining. Our study is intended at evaluating the BER performance of the system when multiple received replica of the STBC transmitted codeword are combined. Accordingly, packet combining is also referred to as code combining and terms are used interchangeably throughout the paper. The BER performance of MIMO STBC has previously been studied considering different modulation schemes and various channel fading models. Two main approaches can be used for this purpose. The first relies on converting the MIMO system model to an equivalent single-input single-output (SISO) model [6] [7], and the second is based on a log-likelihood ratio (LLR) approach [8]. Compared to previous works, the main contribution herein is the performance analysis of MIMO-STBC when ARQ-I is used along with packet combining. Specifically, we combine both methods to present a general analysis for the BER performance of STBC used in conjunction with code combining in MIMO Rayleigh fading channels. Packet combining is also considered in [9] to evaluate the BER performance of 16-QAM in AWGN channel. Compared to the work therein, this paper considers a MIMO-STBC setting under Rayleigh fading and presents an exact analytical expression for the LLRs used in formulating the system's BER considering two combined M-QAM transmissions. Moreover, by means of a simple approximation to the error function erf(.), we provide analytical formulation for the BER resulting from combining more than two transmissions. In the analysis, an LLR-based approach, which is transparent to the considered modulation in each transmission, is adopted. The LLRs derived can be used for soft-input decoding in turbo-coded transmission as in HSDPA [5]. Herein, we consider M-QAM with fixed modulation levels among transmissions. The analysis is presented for the 16-

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QAM case but can straightforwardly be extended to other M-QAM modulations, for coded transmission, and further elaborated for the analysis of the system under consideration when link adaptation, in the form of adaptive modulation and coding [3] and power adaptation [10], is used.

The following content of the paper is structured into five sections. Section II presents the system model. In Section III, we provide a general formulation for the bit LLRs. The probability density functions (PDF) of the aggregate bit LLRs are derived in Section IV. The PDFs are then used in Section V to derive the system's BER. The proposed analytic model is corroborated through simulations in Section VI, followed by concluding remarks drawn in Section VII.

II. System and Channel Models

We consider a wireless communication system with n_T antenna elements at the transmitter and n_R antenna elements at the receiver. The channel is assumed to be a flat slowly-varying Rayleigh fading channel. The complex gain of the link between the i^{th} transmit and j^{th} receive antennas at transmission l, $h_{i,j}^{(l)}$, is defined by a zero-mean circularly symmetric complex Gaussian random variable. Every $4K$ information bits are mapped into symbols $\{s_k^{(l)}\}_{k=1}^K$ which are selected from a Gray-mapping based 16-QAM constellation $\mathcal{C}^{(l)}$ with average energy E_0. Moreover, at each transmission l, $l = 1,\cdots,L$, a constellation rearrangement (CoRe) such as the one used in HSDPA [5] can be utilized, hence the use of the parameter l in the constellation notation $\mathcal{C}^{(l)}$. The resulting symbols $\{s_k^{(l)}\}_{k=1}^K$ are then encoded by a space-time block code defined by a $p \times n_\mathrm{T}$ columnwise orthogonal transmission matrix $\mathcal{G}_{n_\mathrm{T}}$. The entries of matrix $\mathcal{G}_{n_T} = [c_{t,i}]_{t,i=1}^{p,n_\mathrm{T}}$ are linear functions of $\{s_k^{(l)}\}_{k=1}^K$ and their conjugates. At each time slot t, $t = 1,\ldots,p$, the t^{th} row of $\mathcal{G}_{n_\mathrm{T}}$ is transmitted simultaneously through the n_T antennas. The symbol transmission rate R is defined as the ratio between the number of received symbols K and the STBC codeword duration p; i.e., $R = K/p$. For the l^{th} transmission, the signal received by the j^{th} antenna at time t, $t = 1,\ldots,p$, is given by

$$r_{t,j}^{(l)} = \sum_{i=1}^{n_\mathrm{T}} h_{j,i}^{(l)} c_{t,i}^{(l)} + n_{t,j}^{(l)}, \tag{1}$$

where $n_{t,j}^{(l)} \sim \mathcal{N}_c(0,N_0)$ is a Gaussian noise with variance $N_0/2$ per dimension. The signal-to-noise ratio (SNR) per receive antenna is E_s/N_0 where E_s is the average power of the received signal at each receive antenna. Considering a normalized average power over transmit antennas, the average energy of the 16-QAM constellation $\mathcal{C}^{(l)}$ is given by [6]

$$E_0 = 10d^2 = E_s/n_T aR, \tag{2}$$

where $2d$ is the minimum distance between two symbols of $\mathcal{C}^{(l)}$, the same for every transmission l, and a is a constant which depends on the considered STBC matrix $\mathcal{G}_{n_\mathrm{T}}$. Using the SISO equivalency of the MIMO STBC model [6], [7], the received signal can be written as

$$\hat{r}_k^{(l)} = a\|\mathbf{H}^{(l)}\|_\mathrm{F}^2 s_k^{(l)} + \zeta_k^{(l)}, \tag{3}$$

where $\mathbf{H}^{(l)} = [h_{i,j}^{(l)}]_{i,j=1}^{n_\mathrm{R}\times n_\mathrm{T}}$, $\|\cdot\|_\mathrm{F}$ is the matrix Frobenius norm, and $\zeta_k^{(l)} \sim \mathcal{N}_c(0, a\|\mathbf{H}^{(l)}\|_\mathrm{F}^2 N_0)$. Equation (3) can be reformulated as

$$\hat{s}_k^{(l)} = s_k^{(l)} + \eta_k^{(l)}, \tag{4}$$

where $\hat{s}_k^{(l)} = \frac{\hat{r}_k^{(l)}}{a\|\mathbf{H}^{(l)}\|_\mathrm{F}^2}$, and $\eta_k^{(l)} \sim \mathcal{N}_c(0, \frac{N_0}{a\|\mathbf{H}^{(l)}\|_\mathrm{F}^2})$. As can be seen, the equivalent SISO model is an AWGN channel with noise variance equal to $N_0^{(l)} = \frac{N_0}{a\|\mathbf{H}^{(l)}\|_\mathrm{F}^2}$. Finally, defining $\rho^{(l)} \triangleq \|\mathbf{H}^{(l)}\|_\mathrm{F}^2$ which follows a Gamma distribution, the instantaneous SNR per symbol pertaining to the l^{th} transmission is defined by $\gamma^{(l)} = \rho^{(l)} E_s/(n_T R N_0)$ and is distributed according to the PDF

$$f_\gamma(\gamma^{(l)}) = \frac{(\gamma^{(l)})^{n_\mathrm{T}n_\mathrm{R}-1}}{\Gamma(n_\mathrm{T}n_\mathrm{R})\ \overline{\gamma}^{n_\mathrm{T}n_\mathrm{R}}} e^{-\gamma^{(l)}/\overline{\gamma}}, \tag{5}$$

where $\overline{\gamma} \triangleq E_s/(n_\mathrm{T} R N_0)$ and $\Gamma(\cdot)$ is the Gamma function.

In the following, the LLR model is presented considering an AWGN channel, which provides a general representation for MIMO-STBC channel. As previously mentioned, even if the analysis is presented for the 16-QAM case, extension to other square M-QAM modulations can easily be obtained following the procedure described hereafter.

III. Bit LLRs: General Formulation

At transmission l, the LLR $\lambda_{m,k}^{(l)}$ corresponding to bit b_m, $m = 1,2,3,4$, of symbol s_k, $k = 1,\ldots,K$, is defined as [8]

$$\begin{aligned} \lambda_{m,k}^{(l)} &= \log \tfrac{\Pr\{b_m=1|\hat{s}_k,\mathbf{H}\}}{\Pr\{b_m=0|\hat{s}_k,\mathbf{H}\}} \\ &= \log \frac{\sum_{c_1\in S_m^{(1)}} \exp(-\|\hat{s}_k - c_1\|^2/\sigma_{k,l}^2)}{\sum_{c_0\in S_m^{(0)}} \exp(-\|\hat{s}_k - c_0\|^2/\sigma_{k,l}^2)}, \end{aligned} \tag{6}$$

where $S_m^{(0)}$ ($S_m^{(1)}$) is the group of symbols c_0 (c_1) with the m^{th} bit index equal to zero (one), $\sigma_{k,l}^2 = N_0$ in case of AWGN channel, and $\sigma_{k,l}^2 = N_0^{(l)}$ when considering STBC in MIMO Rayleigh fading channels. Using the standard Max-Log approximation, $\log(\sum_i \exp(-X_i)) \approx -\min_j(X_i)$, the bit LLR (6) can be expressed as

$$\lambda_{m,k}^{(l)} = \frac{1}{\sigma_{k,l}^2}\left(\min_{c_0\in S_m^{(0)}} \|\hat{s}_k - c_0\|^2 - \min_{c_1\in S_m^{(1)}} \|\hat{s}_k - c_1\|^2\right). \tag{7}$$

Assuming Gray mapping, demodulation of the Inphase and Quadrature parts of the symbols can be done independently. Moreover, the results over the two dimensions being equivalent, we restrict our analysis to the bits corresponding to the real part of the transmitted symbols (bits at even positions). In this vein, the above bit LLR at positions $m = 2,4$ can be reformulated as [8]:

$$\lambda_{4,k}^{(l)} = \begin{cases} -4\psi_k^{(l)}\hat{s}_{kI} & \text{, if } |\hat{s}_{kI}| \le 2d \\ 8\psi_k^{(l)}(d - \hat{s}_{kI}) & \text{, if } \hat{s}_{kI} > 2d \\ -8\psi_k^{(l)}(d + \hat{s}_{kI}) & \text{, if } \hat{s}_{kI} < -2d \end{cases} \tag{8}$$

and,

$$\lambda_{2,k}^{(l)} = 4\psi_k^{(l)}(|\hat{s}_{kI}| - 2d), \tag{9}$$

where $\hat{s}_{kI} = \Re(\hat{s}_k)$ and $\psi_k^{(l)} = d/\sigma_{k,l}^2$.

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The aggregate LLR resulting from L transmissions of the same symbol is given by

$$\lambda_{m,k}(L) = \sum_{l=1}^{L} \lambda_{m,k}^{(l)}, \tag{10}$$

while when considering CoRe, it is expressed as

$$\lambda_{m,k}(L) = \sum_{l=1}^{L} \pm\lambda_{m^{(l)},k}^{(l)}, \tag{11}$$

where $m^{(l)}$ is the bit's position in transmission l of the bit b_m initially transmitted at position m, and bit LLRs are either subtracted or added to the aggregate LLR, depending on whether their signs are changed or not compared to the first transmission.

IV. Aggregate LLR Distributions

Bit LLRs considering a single transmission follow Gaussian piecewise distributions [9]. Indeed, the bit LLR PDF corresponding to index $m = 4$ of the transmitted symbols s_k, $k = 1, \ldots, K$, is given by

$$p_4(\lambda_{4,k}^{(l)}|s_{kI}, \psi_k^{(l)}) = \begin{cases} \frac{1}{\sqrt{\pi 16 d\psi_k^{(l)}}} \exp\left(-\frac{(\lambda_{4,k}^{(l)}+4d\psi_k^{(l)}s_{kI})^2}{16d\psi_k^{(l)}}\right) & \text{, if } |\lambda_{4,k}^{(l)}| \le 8d\psi_k^{(l)} \\ \frac{1}{\sqrt{\pi 64 d\psi_k^{(l)}}} \exp\left(-\frac{(\lambda_{4,k}^{(l)}+8\psi_k^{(l)}(s_{kI}-d))^2}{64d\psi_k^{(l)}}\right) & \text{, if } \lambda_{4,k}^{(l)} < -8d\psi_k^{(l)} \\ \frac{1}{\sqrt{\pi 64 d\psi_k^{(l)}}} \exp\left(-\frac{(\lambda_{4,k}^{(l)}+8\psi_k^{(l)}(s_{kI}+d))^2}{64d\psi_k^{(l)}}\right) & \text{, if } \lambda_{4,k}^{(l)} > 8d\psi_k^{(l)} \end{cases} \tag{12}$$

and the one corresponding to the bit index $m = 2$ is given by

$$p_2(\lambda_{2,k}^{(l)}|s_{kI}, \psi_k^{(l)}) = \begin{cases} \frac{1}{\sqrt{\pi 16 d\psi_k^{(l)}}} \left[\exp\left(\frac{(\lambda_{4,k}^{(l)}-4\psi_k^{(l)}(s_{kI}-2d))^2}{16d\psi_k^{(l)}}\right) + \exp\left(\frac{(\lambda_{4,k}^{(l)}+4\psi_k^{(l)}(s_{kI}+2d))^2}{16d\psi_k^{(l)}}\right)\right] & \text{, if } \lambda_{2,k}^{(l)} \ge -8d\psi_k^{(l)} \\ 0 & \text{, if } \lambda_{2,k}^{(l)} < -8d\psi_k^{(l)}. \end{cases} \tag{13}$$

Considering independent channels from one transmission to another, the aggregate LLR distribution is equivalent to the convolution of individual LLR distributions defined for each transmission of the same symbol s_k. Moreover, since the effect of CoRe is a simple swapping of bits and/or modification of their signs, the LLR distributions remain Gaussian piecewise functions. Hence, analysis of the LLR distribution is quite the same with or without CoRe. Thus to alleviate notations, we hereafter consider no CoRe. The aggregate bit LLR distribution after L transmissions is then given by

$$p_m(\lambda_{m,k}(L)|s_{kI}) = p_m(\lambda_{m,k}^{(1)}|s_{kI}) \; * \ldots * \; p_m(\lambda_{m,k}^{(L)}|s_{kI}), \tag{14}$$

where $*$ denotes the convolution operator.

To evaluate (14) and provide analytical expressions for the aggregate LLR distributions, we introduce the function $f(z, \mu_1, \mu_2, \vartheta_1, \vartheta_2, r_2, r_1)$ defined as:

$$\begin{aligned} & f(z, \mu_1, \mu_2, \vartheta_1, \vartheta_2, r_2, r_1) \\ &= \int_{r_1}^{r_2} \frac{1}{\pi\sqrt{\vartheta_1\vartheta_2}} e^{-\frac{(x-\mu_1)^2}{\vartheta_1}} e^{-\frac{(z-x-\mu_2)^2}{\vartheta_2}} \mathrm{d}x \\ &= \frac{1}{2\sqrt{\pi(\vartheta_1+\vartheta_2)}} e^{-\frac{(z-(\mu_1+\mu_2))^2}{\vartheta_1+\vartheta_2}} \times \\ &\quad \left[\mathrm{erf}\left(\frac{\vartheta_1(r_2-z+\mu_2)+\vartheta_2(r_2-\mu_1)}{\sqrt{\vartheta_1\vartheta_2(\vartheta_1+\vartheta_2)}}\right) - \mathrm{erf}\left(\frac{\vartheta_1(r_1-z+\mu_2)+\vartheta_2(r_1-\mu_1)}{\sqrt{\vartheta_1\vartheta_2(\vartheta_1+\vartheta_2)}}\right)\right], \end{aligned} \tag{15}$$

where $\mathrm{erf}(\cdot)$ is the error function defined by $\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-x^2}\mathrm{d}x$. The function $f(z, \mu_1, \mu_2, \vartheta_1, \vartheta_2, r_2, r_1)$ can be presented as a *generalized convolution function*. Indeed, the function $f(z, \mu_1, \mu_2, \vartheta_1, \vartheta_2, +\infty, -\infty)$ is equivalent to the standard convolution of two Gaussian functions.

Considering two transmissions ($L = 2$), the aggregate LLR distributions $\lambda_{m,k}(2)$, $m = 2, 4$, are shown in (16) and (17), where for convenience we used the following notations: $\mathbf{U} = \{u_{i,j}\}_{i,j=1}^{4,3}$, $\mathbf{V} = \{v_{i,j}\}_{i,j=1}^{3,2}$ and $\mathbf{W} = \{w_i\}_{i=1}^{2}$ with

$$\mathbf{U} = \begin{bmatrix} -8\psi_k^{(1)}(s_{kI}+d) & -4\psi_k^{(1)}s_{kI} & -8\psi_k^{(1)}(s_{kI}-d) \\ 64d\psi_k^{(1)} & 16d\psi_k^{(1)} & 64d\psi_k^{(1)} \\ -8\psi_k^{(2)}(s_{kI}+d) & -4\psi_k^{(2)}s_{kI} & -8\psi_k^{(2)}(s_{kI}-d) \\ 64d\psi_k^{(2)} & 16d\psi_k^{(2)} & 64d\psi_k^{(2)} \end{bmatrix},$$

$$\mathbf{V} = \begin{bmatrix} 4\psi_k^{(1)}(s_{kI}-2d) & -4\psi_k^{(1)}(s_{kI}+2d) \\ 4\psi_k^{(2)}(s_{kI}-2d) & -4\psi_k^{(2)}(s_{kI}+2d) \\ 16d\psi_k^{(1)} & 16d\psi_k^{(2)} \end{bmatrix},$$

$\mathbf{W} = \begin{bmatrix} -8d\psi_k^{(1)} & -8d\psi_k^{(2)} \end{bmatrix}$, and we assumed $w_1 > w_2$.

These PDFs are represented by piecewise functions formed by the sum of $f(z, \mu_1, \mu_2, \vartheta_1, \vartheta_2, r_2, r_1)$ terms. As it can be seen, the PDF formulae for $L = 2$ are more complicated compared to the piecewise Gaussian distribution resulting when a single transmission is performed ($L = 1$). In addition, analytical formulation of the LLR distributions in closed-form is untractable due to the error function term involved in the function $f(z, \mu_1, \mu_2, \vartheta_1, \vartheta_2, r_2, r_1)$. To tackle this problem, we use a simple approximation for the Gaussian distribution function [11] to express the error function erf(·) as piecewise polynomial function according to:

$$\mathrm{gerf}(x) = \begin{cases} -(0.4x^2 - 1.24|x|)\,\mathrm{sign}(x) & \text{if } |x| \le 1.556, \\ 0.98\,\mathrm{sign}(x) & \text{if } 1.556 < |x| < 1.838, \\ \mathrm{sign}(x) & \text{if } 1.838 \le |x|, \end{cases} \tag{18}$$

where $|\cdot|$ denotes the absolute value operator. By means of this approximation, the aggregate LLR distributions considering two transmissions, can be written in closed-form as the sum of piecewise functions of the form $q_j(x) = x^j e^{-(\alpha x+\beta)^2}$, $j \in \mathbb{N}$.

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Now, in order to compute the aggregate LLR distributions $p_m(\lambda_{m,k}(3)|s_{kI}, \psi_k^{(1)}, \psi_k^{(2)}, \psi_k^{(3)})$ when three transmissions are used, the primitive of $q_j(x)$ is needed. The latter is given by

$$Q_j(x) = \int x^j e^{-(\alpha x+\beta)^2} \mathrm{d}x, \tag{19}$$

where α and β are functions of s_{kI}, d and $\left\{\psi_k^{(1)}, \ldots, \psi_k^{(L)}\right\}$. Using integration by part, (19) can be expressed as

$$Q_j(x) = P_j(x) \cdot e^{-(\alpha x+\beta)^2} + C_j \cdot \mathrm{erf}(\alpha x + \beta), \tag{20}$$

where $P_j(x)$ is a polynomial function of degree $(j-1)$, except for $j = 0$ where it is equal to zero, C_j is a constant independent of x, and we have

$$2\alpha^2 P_{j+2}(x) = (j+1)P_j(x) - 2\alpha\beta P_{j+1}(x) - x^{j+1}, \tag{21}$$

$$2\alpha^2 C_{j+2} = (j+1)C_j - 2\alpha\beta C_{j+1}, \tag{22}$$

with,

$$\begin{array}{ll} P_0 = 0 & , \quad P_1 = -1/2\alpha^2, \\ C_0 = \sqrt{\pi}/2\alpha & , \quad C_1 = -\beta\sqrt{\pi}/2\alpha^2. \end{array} \tag{23}$$

The aggregate LLR distributions for $L = 3$ are piecewise functions composed by the sum of terms of the form $Q_j(x)$ (20). Thus, using the latter jointly with the definitions of $P_j(x)$ and $C_j(x)$, an analytic formulation of the PDFs $p_m(\lambda_{m,k}(3)|s_{kI}, \psi_k^{(1)}, \psi_k^{(2)}, \psi_k^{(3)})$, $m = 2, 4$, can easily be obtained.

The LLR distributions considering more than three transmissions can subsequently be deduced. Indeed, using (18), (20) becomes equivalent to a sum of terms in the form of $q_j(x)$. Therefore, the analysis used for $L = 3$ can be applied for the general case in order to provide an analytic formulation of the aggregate LLR PDFs $p_m(\lambda_{m,k}(L)|s_{kI}, \psi_k^{(1)}, \ldots, \psi_k^{(L)})$, $m = 2, 4$, for L combined transmissions.

V. BER Derivation

In the case of AWGN channel, $\psi_k^{(l)} = d/N_0$ for $l = 1, \cdots, L$, hence, the aggregate LLR distributions $p_m(\lambda_{m,k}(L)|s_k, \psi_k^{(1)}, \ldots, \psi_k^{(L)})$ can be denoted by $p_m(\lambda_{m,k}(L)|s_k)$. The BER considering M-QAM and using L transmissions can then be expressed as

$$P_\mathrm{e}^{\mathrm{AWGN},L} = \frac{1}{\log_2(M)} \sum_{m=1}^{\log_2(M)} P_\mathrm{e}^{\mathrm{AWGN},L}(m), \tag{24}$$

where

$$P_\mathrm{e}^{\mathrm{AWGN},L}(m) = \frac{1}{M}\left[\sum_{s_k \in S_m^{(1)}} \int_{-\infty}^{0} p_m(\lambda_{m,k}(L)|s_k)\mathrm{d}\lambda_{m,k}(L) + \sum_{s_k \in S_m^{(0)}} \int_{0}^{\infty} p_m(\lambda_{m,k}(L)|s_k)\mathrm{d}\lambda_{m,k}(L)\right]. \tag{25}$$

Taking into consideration the fact that the LLR PDFs for L combined transmissions are piecewise functions defined by the sum of terms of the form $q_j(x)$, the analytical expression for the BER can then be formulated using (18)-(23). On the other hand, when using STBC in MIMO configurations over Rayleigh fading channels, the parameter $d\psi_k^{(l)}$ is given by

$$d\psi_k^{(l)} = d^2/N_0^{(l)} = E_s\rho^{(l)}/\phi n_\mathrm{T} R N_0 = \gamma^{(l)}/\phi, \tag{26}$$

where ϕ is a modulation-dependent constant, for instance $\phi = 10$ in the 16-QAM case (2). Accordingly, the LLR distributions $p_m(\lambda_{m,k}(L)|s_k, \psi_k^{(1)}, \ldots, \psi_k^{(L)})$ can be denoted as $p_m(\lambda_{m,k}(L)|s_k, \gamma^{(1)}, \ldots, \gamma^{(L)})$. Using the derived PDFs, formulation of the BER for the STBC system with packet combining in MIMO Rayleigh fading channels, $P_\mathrm{e}^{\mathrm{STBC},L}$, can then be deduced using (5) and (24) in the following expression

$$P_\mathrm{e}^{\mathrm{STBC},L} = \int_0^\infty \cdots \int_0^\infty P_\mathrm{e}^{\mathrm{AWGN},L} f_\gamma(\gamma^{(1)}) \ldots f_\gamma(\gamma^{(L)}) \mathrm{d}\gamma^{(1)} \ldots \mathrm{d}\gamma^{(L)}. \tag{27}$$

VI. Numerical Results

Sample numerical results are now presented to illustrate the BER performance of the system under study. Considering 16-QAM, BER results are shown as a function of the SNR per receive antenna, E_s/N_0, and comparisons between analytical and simulation results are carried out to show the validity of the proposed analytical model. In order to get accurate estimation for the simulated BER, more than 3 10^6 bits were generated to compute the BER statistics. For the AWGN channel case, analytical results are computed through direct use of (24) and (25). As for the STBC case over MIMO Rayleigh fading channels, in addition to the latter expressions, we use the Gauss-Legendre Quadrature rule [12] for the evaluation of the integral involved in the BER computation (27). Fig. 1 and Fig. 2 present the BER performance when packet combining is used for $L = 2, 3, 4$ retransmissions in an AWGN channel with and without CoRe, respectively. The reference case pertaining to no packet combining ($L = 1$) is also plotted. In Fig. 2, the CoRe used prior to symbol modulation through 16-QAM corresponds to the one adopted in HSDPA [5]. As observed in both cases, the analytical and simulation results are almost in perfect match. Comparison of both figures also illustrates the gain obtained through CoRe. For the purpose of illustrating the validity of our approach under Rayleigh fading, we assume no CoRe for the MIMO-STBC case and present the BER performance for different STBC mappings. In particular, in Figs 3 and 4, MIMO configurations with $n_\mathrm{T} = n_\mathrm{R} = 2$ and $n_\mathrm{T} = n_\mathrm{R} = 3$ are respectively considered. Depending on the value of n_T, the space-time block code used corresponds either to the mapping provided in [1], namely, $\mathcal{G}_2$ with $a = 1$ and $R = 1$, or the half-rate code $\mathcal{G}_3$ with $a = 2$ and $R = 1/2$ provided in [2]. Similar to the AWGN case, we observe that for different numbers of combined transmissions, the analytical results are almost in perfect match with the simulation results, indicating the validity of the analytical model and the accuracy of the approximation used for the erf(.) function.

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VII. Conclusion

We considered a MIMO system utilizing space-time block coding and adopted an LLR approach to present a BER analysis when M-QAM modulation is used. Considering that the LLRs follow piecewise Gaussian distributions in AWGN channel, and adopting a simple approximation for the error function erf$(\cdot)$, we presented an analytical formulation for the LLR distributions when packet retransmission in the form of ARQ-Type I protocol is used along with packet combining. The analysis considers 16-QAM used with or without constellation rearrangement, is performed for AWGN and MIMO Rayleigh fading channels, and can easily be extended to other M-QAM modulations. BER numerical results and comparisons are provided and show the accuracy of the proposed analytical model.

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$$p_4(\lambda_{4,k}(2)|s_{kI},\psi_k^{(1)},\psi_k^{(2)}) = \begin{cases} f(\lambda_{4,k}(2),u_{1,3},u_{3,1},u_{2,3},u_{4,1},\lambda_{4,k}(2)-w_2,-\infty) + f(\lambda_{4,k}(2),u_{1,3},u_{3,2},u_{2,3},u_{4,2},\lambda_{4,k}(2)+w_2,\lambda_{4,k}(2)-w_2) \\ + f(\lambda_{4,k}(2),u_{1,3},u_{3,3},u_{2,3},u_{4,3},-w_1,\lambda_{4,k}(2)+w_2) + f(\lambda_{4,k}(2),u_{1,2},u_{3,3},u_{2,2},u_{4,3},w_1,-w_1) \\ + f(\lambda_{4,k}(2),u_{1,1},u_{3,3},u_{2,1},u_{4,3},+\infty,w_1) & \text{, if } \lambda_{4,k}(2) \le -(w_1+w_2) \\ f(\lambda_{4,k}(2),u_{1,3},u_{3,1},u_{2,3},u_{4,1},\lambda_{4,k}(2)-w_2,-\infty) + f(\lambda_{4,k}(2),u_{1,3},u_{3,2},u_{2,3},u_{4,2},-w_1,\lambda_{4,k}(2)-w_2) \\ + f(\lambda_{4,k}(2),u_{1,2},u_{3,2},u_{2,2},u_{4,2},\lambda_{4,k}(2)+w_2,-w_1) + f(\lambda_{4,k}(2),u_{1,2},u_{3,3},u_{2,2},u_{4,3},w_1,\lambda_{4,k}(2)+w_2) \\ + f(\lambda_{4,k}(2),u_{1,1},u_{3,3},u_{2,1},u_{4,3},+\infty,w_1) & \text{, if } -(w_1+w_2) < \lambda_{4,k}(2) \le -(w_1-w_2) \\ f(\lambda_{4,k}(2),u_{1,3},u_{3,1},u_{2,3},u_{4,1},-w_1,-\infty) + f(\lambda_{4,k}(2),u_{1,2},u_{3,1},u_{2,2},u_{4,1},\lambda_{4,k}(2)-w_2,-w_1) \\ + f(\lambda_{4,k}(2),u_{1,2},u_{3,2},u_{2,2},u_{4,2},\lambda_{4,k}(2)+w_2,\lambda_{4,k}(2)-w_2) + f(\lambda_{4,k}(2),u_{1,2},u_{3,3},u_{2,2},u_{4,3},w_1,\lambda_{4,k}(2)+w_2) \\ + f(\lambda_{4,k}(2),u_{1,1},u_{3,3},u_{2,1},u_{4,3},+\infty,w_1) & \text{, if } -(w_1-w_2) < \lambda_{4,k}(2) \le (w_1-w_2) \\ f(\lambda_{4,k}(2),u_{1,3},u_{3,1},u_{2,3},u_{4,1},-w_1,-\infty) + f(\lambda_{4,k}(2),u_{1,2},u_{3,1},u_{2,2},u_{4,1},\lambda_{4,k}(2)-w_2,-w_1) \\ + f(\lambda_{4,k}(2),u_{1,2},u_{3,2},u_{2,2},u_{4,2},w_1,\lambda_{4,k}(2)-w_2) + f(\lambda_{4,k}(2),u_{1,1},u_{3,2},u_{2,1},u_{4,2},\lambda_{4,k}(2)+w_2,w_1) \\ + f(\lambda_{4,k}(2),u_{1,1},u_{3,3},u_{2,1},u_{4,3},+\infty,\lambda_{4,k}(2)+w_2) & \text{, if } (w_1-w_2) < \lambda_{4,k}(2) \le (w_1+w_2) \\ f(\lambda_{4,k}(2),u_{1,3},u_{3,1},u_{2,3},u_{4,1},-w_1,-\infty) + f(\lambda_{4,k}(2),u_{1,2},u_{3,1},u_{2,2},u_{4,1},w_1,-w_1) \\ + f(\lambda_{4,k}(2),u_{1,1},u_{3,1},u_{2,1},u_{4,1},\lambda_{4,k}(2)-w_2,w_1) + f(\lambda_{4,k}(2),u_{1,2},u_{3,1},u_{2,2},u_{4,1},\lambda_{4,k}(2)+w_2,\lambda_{4,k}(2)-w_2) \\ + f(\lambda_{4,k}(2),u_{1,1},u_{3,3},u_{2,1},u_{4,3},+\infty,\lambda_{4,k}(2)+w_2) & \text{, if } (w_1+w_2) < \lambda_{4,k}(2). \end{cases} \tag{16}$$

$$p_2(\lambda_{2,k}(2)|s_{kI},\psi_k^{(1)},\psi_k^{(2)}) = \begin{cases} f(\lambda_{2,k}(2),v_{1,1},v_{2,1},v_{3,1},v_{3,2},\lambda_{2,k}(2)+w_2,-w_1) + f(\lambda_{2,k}(2),v_{1,1},v_{2,2},v_{3,1},v_{3,2},\lambda_{2,k}(2)+w_2,-w_1) \\ + f(\lambda_{2,k}(2),v_{1,2},v_{2,1},v_{3,1},v_{3,2},\lambda_{2,k}(2)+w_2,-w_1) + f(\lambda_{2,k}(2),v_{1,2},v_{2,2},v_{3,1},v_{3,2},\lambda_{2,k}(2)+w_2,-w_1) \\ & \text{, if } -(w_1+w_2) \le \lambda_{2,k}(2) \\ 0 & \text{, if } \lambda_{2,k}(2) < -(w_1+w_2). \end{cases} \tag{17}$$

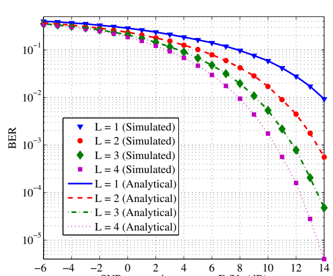

Fig. 1. Comparison between analytical and simulation results considering AWGN channel and up to $L = 4$ combined transmissions

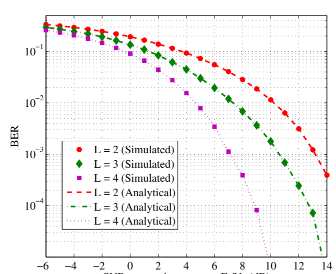

Fig. 2. Comparison between analytical and simulation results considering AWGN channel and up to $L = 4$ transmissions with constellation rearrangement

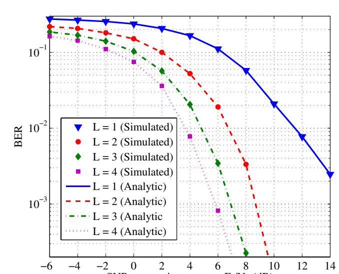

Fig. 3. Comparison between analytical and simulation results using STBC in 2×2 MIMO Rayleigh fading channels and up to $L = 4$ combined transmissions

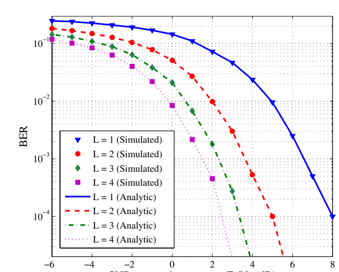

Fig. 4. Comparison between analytical and simulation results, using STBC in 3×3 MIMO Rayleigh fading channels and up to $L = 4$ combined transmissions